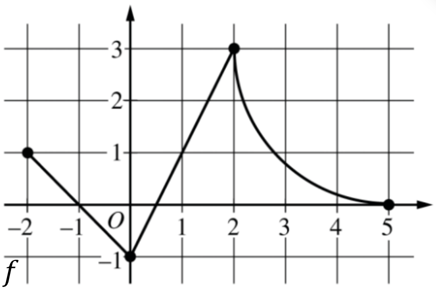
Sample Question 1

Allotted time: 25 minutes (plus 5 minutes to submit)

t (hours)	0	0.3	1	2.8	4
$v_p(t)$ (meters per hour)	0	55	-29	55	48



Graph of f

The velocity of a particle, P, moving along the x-axis is given by the differentiable function $v_P(t)$, where $v_P(t)$ is measured in meters per hour and t is measured in hours. Selected values of $v_P(t)$ are shown in the table above. Particle P is at the origin at time $t = 0$. The acceleration of particle P, $a_P(t)$, at $t = 1$ is known to be $a_P(1) = -10$.

Also, the continuous function f is defined on the closed interval $-6 \le t \le 5$. The figure above shows a portion of the graph of f, consisting of two line segments and a quarter of a circle centered at the point $(5, 3)$. It is known that the point $(3, 3 - \sqrt{5})$ is on the graph of f.

(a) Find $\frac{d}{dt}[f(t) \cdot v_P(t)]|_{t=1}$

(b) Use a trapezoidal sum with the three subintervals $[0, 0.3]$, $[0.3, 1]$, and $[1, 2.8]$ to approximate the value of $\int_0^{2.8} v_p(t)\,dt$.

(c) If $\int_{-6}^{5} f(t)\,dt = 7$, find the value of $\int_{-6}^{-2} f(t)\,dt$. Show the work that leads to your answer.

(d) Evaluate $\int_3^5 (2f'(t) + 4)dt$.

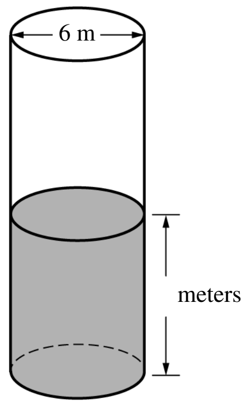
(e) The function g is given by $g(t) = \int_{-2}^{t} f(x)\,dx$. Find the absolute maximum of g on the interval $-2 \le x \le 5$. Justify your answer.

(f) Using $g(t)$ from part (e), is the rate of change in g increasing or decreasing at $t = 3$? Explain your reasoning.

(g) Find $\lim_{t \to 1} \dfrac{e^t - 3f(t)}{v_P(t) - \cos(\pi t)}$.

Sample Question 2

Allotted time: 15 minutes (plus 5 minutes to submit)



A cylindrical barrel with a diameter of 6 meters contains collected rainwater, as shown in the figure above. The water drains out through a valve (not shown) at the bottom of the barrel. The rate of change of the height h of the water in the barrel with respect to time t is modeled by $\frac{dh}{dt} = -\frac{1}{5}\sqrt{h}$, where h is measured in meters and t is measured in seconds. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

(a) Find the rate of change of the volume of water in the barrel with respect to time when the height of the water is 10 meters. Indicate units of measure.

(b) When the height of the water is 8 meters, is the rate of change of the height of the water with respect to time increasing or decreasing? Explain your reasoning.

(c) At time $t = 0$ seconds, the height of the water is 16 meters. Use separation of variables to find an expression for h in terms of t.